Physics Unit 6: Rotational Inertia Math Problems

Calculating Rotational Inertia
When all the mass \( m \) of an object is concentrated at the same distance \( r \) from a rotational axis, then the rotational inertia is,

\[
I = mr^2
\]

In reality, the mass of an object is more spread out and the rotational inertia is less and the formula is different. An object’s rotational inertia depends on its mass, dimensions and shape. Even for the same mass, dimensions and shape the rotational inertia is different for the different axes of rotation. Here are the equations for some common shapes and axes of rotation:

Rotational Inertia, Torque and Angular Acceleration
Just like translational inertia (aka mass), force and acceleration are related to each other by Newton’s Second Law of Motion:

\[
F_{net} = ma
\]

so too are the rotational counterparts related to each other:

\[
\tau_{net} = I\alpha
\]

1. What is the inertia of a hoop rotating about its symmetry axis if its mass is 2 kg and its radius is 0.4 m?

2. What is the inertia of a hoop rotating about its central diameter if its mass is 2 kg and its radius is 0.4 m?
3. A hoop will have a smaller / larger rotational inertia rotating around its central diameter compared to rotating around its symmetry axis.

4. What is the inertia of a solid cylinder rotating about its symmetry axis if its mass is 0.50 kg, its radius is 0.25m and its length is 1.0 m?

5. What is the inertia of a solid cylinder rotating about its central diameter if its mass is 0.50 kg, its radius is 0.25m and its length is 1.0 m?

6. A solid cylinder will have a smaller / larger rotational inertia rotating around its symmetry axis compared to rotating around its central diameter.

7. What is the inertia of a rod rotating about its central diameter if its mass is 0.50 kg and its length is 1.0 m?

8. What is the inertia of a rod rotating about its end if its mass is 0.50 kg and its length is 1.0 m?

9. A rod will have a smaller / larger rotational inertia rotating around its end compared to rotating around its central diameter.

10. What is the rotational inertia of a 0.5 kg solid metal sphere that has a diameter of 10 cm?
11. What is the rotational inertia of a 0.5 kg hollow metal sphere that has a diameter of 10 cm?

12. For the same mass and radius you would expect a solid sphere to have a smaller/larger rotational inertia than a hollow sphere.

13. For a hollow sphere and solid sphere made of the same material and with the same radius you would expect the hollow/solid sphere to have the larger rotational inertia.

14. For the same mass and radius would you expect a pipe (aka hollow cylinder) to have a smaller/larger rotational inertia than a solid cylinder?

15. For a hollow cylinder and a solid cylinder made of the same material and with the same radius and length you would expect the hollow/solid cylinder to have the larger rotational inertia.

16. For the same applied torque, which movement will produce the fastest rotation? Assume the same person is performing all three movements.
   a. An ice skater spinning vertically with their arms tight to their chest.
   b. A gymnast doing a summersault in the tucked position?
   c. A gymnast doing a summersault in the laid out position?
   d. A gymnast doing a giant circle on the high bar?

17. What is the angular acceleration if a force of 3 N is applied perpendicular to the radius of a 5 kg solid sphere that has a radius of 0.6 meters?

18. What is the angular acceleration if a force of 3 N is applied perpendicular to the radius of a 5 kg hollow sphere that has a radius of 0.6 meters?
19. What force must be applied perpendicular to the end of a rod to generate an angular acceleration of $2\pi \text{ rad/s}^2$ around the central diameter (longitudinal axis)?

20. What force must be applied perpendicular to the end of a rod to generate an angular acceleration of $2\pi \text{ rad/s}^2$ around the other end (medial axis) of the rod?

21. What is the radius and rotational inertia of a 20g ring (hoop) if a 0.005N force applied perpendicular to the radius generates an angular acceleration of 25.0 rad/s$^2$ about the central diameter (longitudinal axis)? [Hints: watch your units and you’ll need two equations to solve for two unknowns.]